HANOI DEPARTMENT OF EDUCATION AND TRAINING

**ME LINH AND SOC SON DISTRICTS**

*(The test consists of 02 pages)*

**HANOI OPEN MATHEMATICAL COMPETITION**

**SENIOR SECTION – YEAR 2019**

*Exam time: 120 minutes*

**Date: 11 January 2019**

- Examinee's full name: ........................................................
- Registration number:.......................... Room: .....................
- Important: Write your answers in the exam papers provided.

**Part I.** (10.0 marks)

*Questions 1 - 10 are short questions, each worth 1 mark, and you can answer without showing your working.*

**Question 1.** Let $\{x_n\}$ be a sequence given by

$$\begin{cases} x_1 = \sqrt{6} \\ x_{n+1} = \sqrt{6 + x_n}, n \geq 1 \end{cases}$$

Find $[x_{2019}]$ (where $[x]$ is the Greatest Integer Function of $x$).

**Question 2.** For which values of $m$, the equation

$$x^2 - (2m+1)x + m^2 + 1 = 0$$

has two real solutions $x_1, x_2$ such that $x_1 = 2x_2$?

**Question 3.** Suppose that $x + y = 1$. Evaluate $x^3 + y^3 + 3xy$.

**Question 4.** Solve the inequality $3|2x-1| < 2x+1$.

**Question 5.** Evaluate $\left(4+\sqrt{15}\right)\left(\sqrt{10}-\sqrt{6}\right)\sqrt{4-\sqrt{15}}$.

**Question 6.** If $2x^2 + 3y^2 \leq 5$, find the sum of the maximum value and the minimum value attained by $2x + 3y$.

**Question 7.** $n$ is the largest positive integer such that $n^3 + 100$ is divisible by $n + 10$. Find the digit sum of $n$.

**Question 8.** Let $a$, $b$ and $c$ be real and positive parameters. How many solutions does the following equation have?

$$\sqrt{a+bx} + \sqrt{b+cx} + \sqrt{c+ax} = \sqrt{b-ax} + \sqrt{c-bx} + \sqrt{a-cx}.$$

**Question 9.** Let $\{x_n\}$ be a sequence defined by

$$\begin{cases} x_0 = 3 \\ x_1 = 4 \\ x_{n+1} = x_{n-1}{}^2 - nx_n \forall n \geq 1. \end{cases}$$

Then $x_{2019} = ?$

**Question 10.** Given the real numbers *a*, *b*, *c*, *d* and *e* satisfy the relations $a+b+c+d+e=8$ and $a^2+b^2+c^2+d^2+e^2=16.$

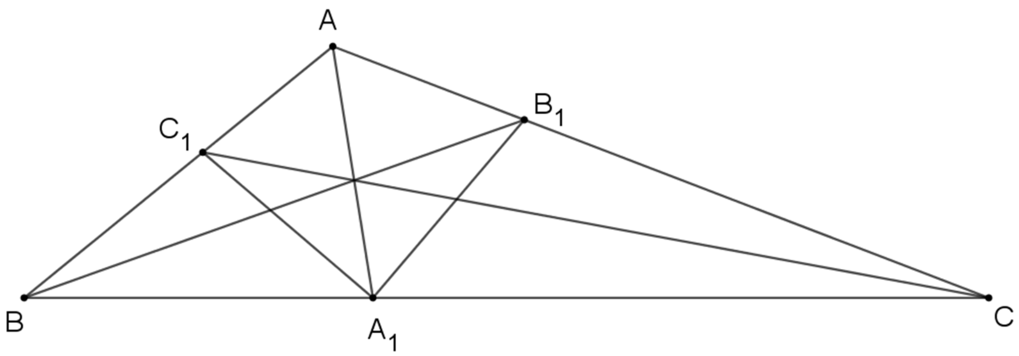
Determine the sum of the maximum value and the minimum value of *a*.

**Part II.** (10.0 marks)

*Questions 11 - 15 are longer questions, each worth 2 marks, and you have to show your working.*

**Question 11.** Prove that $\sin 10°$ is an irrational number.

**Question 12.** Consider a triangle $\Delta ABC$, $\widehat{BAC} = 120°$. Let $AA_1, BB_1, CC_1$ be three angle bisectors of $\Delta ABC$ ($A_1 \in BC, B_1 \in AC, C_1 \in AB$). Prove that $\widehat{B_1A_1C_1} = 90°$.




**Question 13.** Determine the number of ways to choose 5 numbers from the first 18 positive integers such that any two chosen numbers differ by at least 2.

**Question 14.** Solve the equation

$$(x+3)^3 - (x+1)^3 = 56$$

**Question 15.** Prove that

$$16 < \sum_{k=1}^{80} \frac{1}{\sqrt{k}} < 17$$

*The end.*